Applications of Computer Algebra (ACA) 2026

Faculty of Mathematical-Natural Sciences, University of Prishtina

CONFERENCE PROGRAM

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Schedule

Monday, June 1 — Room 153

Time	Event
08:30–09:00	Registration
09:00–09:15	Conference OPENING
09:30–10:00	**Mariya Dzhumalieva-Stoeva** (SS4) *Construction of Entanglement-Assisted Quantum Error-Correcting Codes from Classical Linear Codes with Various Hull Dimensions*
10:00–10:30	**Stela Zhelezova** (SS4) *The Deficiency Spread of a Transitive Deficiency One Parallelism of* $PG(5,2)$
10:30–11:00	**Buket Özkaya** (SS4) *On Schur Product of Algebraic Codes*
11:00–11:30	Coffee break
11:30–12:00	**Talha Arıkan** (SS4) *Some Asymptotically Optimal AAD Families*
12:00–12:30	**Ferruh Özbudak** (SS4) *Binary Codes from Kronecker Product of Matrices*
12:30–13:30	Lunch break
13:30–14:30	**David Kalaj** (Plenary) *The norm of Cauchy transform*
14:30–15:00	**David Jeffrey** (SS3) *Diagonalization and Change of Basis in Linear Algebra*
15:00–15:30	Coffee break
15:30–16:00	**Michel Beaudin** (SS3) *Be careful when adding an integration constant*
16:00–16:30	**José Manuel Dos Santos** (SS3) *Computer Algebra Systems in Secondary Mathematics Education: The Role of GeoGebra CAS View and Generative Artificial Intelligence in Teaching and Learning Mathematics*
19:00	Conference Dinner

Tuesday, June 2 — Room 153

Time	Event
09:00–10:00	Coffee with the Rector of the University, Prof. Hajrullahu
10:00–10:30	**Alexandre V. Morozov** (SS2) *Evolutionary emergence of cooperativity in the game of Prisoner's Dilemma*
10:30–11:00	**Atsushi Mochizuki** (SS2) *Topological Origin of Modularity in Chemical Reaction Networks*
11:00–11:30	**Mashaal Nageen** (SS2) *Structural identifiability of bacteria co-culturing models*
11:30–12:00	**Francesco Nowell** (SS2) *Polyhedral Geometry of Max-Linear Bayesian Networks*
12:00–12:30	**Murad Banaji** (SS2) *Computer Algebra to Study Bifurcations in Chemical Reaction Networks*

12:30–13:30	Lunch break
13:30–14:30	**Stephen M. Watt** (Plenary)
14:30–15:00	**Dijana Oreški** (SS1) *Evaluating Large Language Models on Symbolic Reasoning Tasks*
15:00–15:30	Coffee break
15:30–16:00	**Florian Hellwig** (SS1) *Resolving the Remaining Cases of Carlitz-Extensiveness over Finite Fields using SageMath*
16:00–16:30	**Arber Selimi** (SS1) *Nonabelian Cohomology and Classifying Spaces of Crossed Complexes*

Tuesday, June 2 — Room 114 (Parallel Session)

Time	Event
10:30–11:00	**María Cumplido** (SS5) *Classification of Artin groups that admit retractions to parabolic subgroups*
11:00–11:45	**Giovanni Sartori** (SS5) *Artin groups and their isomorphism problem*
11:45–12:30	**Oli Jones** (SS5) *JSJ decompositions of Artin groups*
12:30–13:30	Lunch break
14:30–15:00	**Federica Gavazzi** (SS5) *Parabolic Subgroups and the Word Problem in Virtual Artin Groups*
15:00–15:30	Coffee break
15:30–16:00	**Davide Spriano** (SS5) *A finitely presented group is almost planar if and only if it is virtually planar*
16:00–16:30	**José Manuel Dos Santos** (SS3) *From Curricular Signals to Exploratory Tasks: A Curriculum and Pedagogical Design Case Study for Integrating CAS in Upper Secondary Education*

Wednesday, June 3

Time	Event
08:00	Excursion (Lunch will be provided)

Thursday, June 4 — Room 153

Time	Event
09:00–09:30	**Patrick Browne and Padraig Ó Catháin** (SS1) *Strongly Orthogonal Subsets in Exceptional Root Systems: Theory and Computation*
09:30–10:00	**Yang Zhang** (SS1) *Efficient Algorithms for Quaternion Tensor Completion*
10:00–10:30	**Masaki Suzuki** (SS3) *Integration of Computer Algebra Systems and AI in HTML-based Mathematics Materials using KeTCindyJS*

10:30–11:00	**Eli Bagno** (SS3)
	Envelopes of Circles Centered on a Cubic-Quadratic: A Dynamic Exploration of their Topology and Singular Locus
11:00–11:30	Coffee break
11:30–12:00	**Eva Dimitrova & Elena Varbanova** (SS3)
	ChatGPT as an instrument for enhancement of learners' interest in undergraduate mathematics
12:00–12:30	**José Manuel Dos Santos** (SS3)
	Intersections of a Torus with Cubic Surfaces via Planar Section Reduction
12:30–13:30	Lunch break
13:30–14:30	**Setsuo Takato** (SS3)
	Solving Wasan Problems on Malfatti Circles via the mnr Method: Human-AI Collaborative Automated Derivation using KeTCindy
14:30–15:00	**Thierry Dana-Picard** (SS3)
	Ovals and pedal curves: space trajectory models and genealogy of plane curves
15:00–15:30	Coffee Break with the Dean of FMNS, Prof. Bllaca

Friday, June 5 — Room 153

Time	Event
09:30–10:00	**Uran Meha** (SS1)
10:00–10:30	**Alban Kryeziu** (SS1)
10:30–11:00	**Cameron Calk** (SS1)
11:00–11:30	Coffee break
11:30–12:00	**Islam Foniqi** (SS1)
	A Journey Through Decision Problems in Monoids and Groups
12:00–12:30	Conference CLOSING

Invited Speakers

David Kalaj is a distinguished mathematician from the University of Montenegro whose work has made a significant impact across several areas of modern mathematical analysis. His research interests have included complex analysis, harmonic analysis, differential geometry, functional analysis, and partial differential equations, reflecting both the breadth and depth of his scientific contributions. Among other results, he solved the Gaussian curvature conjecture, a longstanding mathematical problem that had remained open for more than seventy years. Prof. Dr. Kalaj has published around 130 papers indexed in SCIE and has received approximately 2,000 citations. In recognition of his outstanding achievements, he has received some of the most prestigious awards in Montenegro, including the Award of 13 July.

Stephen M. Watt is Professor in the David R. Cheriton School of Computer Science at the University of Waterloo and Distinguished University Professor Emeritus of the University of Western Ontario. His research is in computer algebra and symbolic mathematical computation, broadly understood: from algebraic algorithms and symbolic-numeric methods, through software systems and programming languages for mathematical computation, to mathematical knowledge management, user interfaces, and mathematical handwriting recognition.

Watt was one of the original authors of the Maple computer algebra system at Waterloo and the Axiom system at IBM, and principal architect of the Aldor programming language, originally known as A#. Aldor was designed to support efficient symbolic and numeric mathematical software through rich type structures and high-performance implementation. He was also one of the authors of MathML and InkML, the W3C standards for mathematical notation and digital ink.

From 1997 to 2015, Watt was Professor of Computer Science at Western University, where he served as Chair of Computer Science and founding director of the Ontario Research Centre for Computer Algebra. From 2015 to 2020, he served as Dean of the Faculty of Mathematics at Waterloo. He has authored more than 280 publications and has served as a director or board chair of several organizations, including Maplesoft, the Descartes Systems Group (NASDAQ: DSGX), the Numerical Algorithms Group, the Fields Institute, and the International Mathematical Knowledge Trust.

Participants

First name	Last name	University or organization
Alban	Kryeziu	University of Groningen
Alexandre	Morozov	Rutgers University
AmirHosein	Sadeghimanesh	Coventry University
Arber	Selimi	University of Prishtina
Artan	Alidema	University of Prishtina
Artan	Berisha	University of Prishtina
Atsushi	Mochizuki	Kyoto University
Behar	Baxhaku	University of Prishtina
Bujar	Fejzullahu	University of Prishtina
Buket	Özkaya	Middle East Technical University
Cameron	Calk	Aix-Marseille Université
Daniela	Nikolova	Florida Atlantic University
David	Jeffrey	University of Western Ontario
David	Kalaj	University of Montenegro
Davide	Spriano	University of Warwick
Dijana	Oreski	University of Zagreb
Edmond	Aliaga	University of Prishtina
Elena	Varbanova	Technical University of Sofia
Elver	Bajrami	University of Prishtina
Federica	Gavazzi	Heriot-Watt University, Edinburgh
Florian	Hellwig	University of Zurich
Francesco	Nowell	TU Berlin
Giovanni	Sartori	Heriot-Watt University
Islam	Foniqi	Technical University of Berlin
José Manuel	Dos Santos	University of Coimbra
Kajtaz	Bllaca	University of Prishtina
Kamel	Al-Khaled	Jordan University of Science and Technology
María	Cumplido	Universidad de Sevilla
Mariya	Dzhumalieva-Stoeva	St. Cyril and St. Methodius University of Veliko Tarnovo, Bulgaria
Masaki	Suzuki	National Institute of Technology, Numazu College
Mashaal	Nageen	Coventry University, UK
Michael	Wester	University of New Mexico
Michel	Beaudin	École de technologie supérieure
Murad	Banaji	Lancaster University, UK
Nazahet	Fellah	University M'hamed Bougara of Boumerdès, Algeria
Oli	Jones	Technische Universität Berlin
Patrick	Browne	Technological University of the Shannon
Peter	Boyvalenkov	IMI, Bulgarian Academy of Sciences
Qendrim	Gashi	UP and UMD
Setsuo	Takato	KeTCindy Center
Stela	Zhelezova	Institute of Mathematics and Informatics, BAS
Stephen	Watt	University of Waterloo
Talha	Arıkan	Hacettepe University
Tefjol	Pllaha	University of South Florida
Thierry	Dana-Picard	Jerusalem College of Technology
Uran	Meha	Independent
Valona	Lahu	University of Prishtina
Yang	Zhang	University of Manitoba, Canada

Abstracts

The abstracts below are ordered chronologically according to the conference schedule. Within each day, they are grouped by room and session.

Monday, June 1 — Room 153

Coding Theory and Its Applications (SS4)

Construction of Entanglement-Assisted Quantum Error-Correcting Codes from Classical Linear Codes with Various Hull Dimensions

Stefka Bouyuklieva[1], Mariya Dzhumalieva-Stoeva[1*]
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Key words: *entanglement-assisted quantum error-correcting codes; classical linear codes; hull dimensions*

We present results on the construction of entanglement-assisted quantum error-correcting codes derived from classical linear codes with various hull dimensions. The parameters of these quantum codes depend not only on the length, dimension, and minimum distance of the underlying classical codes, but also on the dimension of their hulls.

Let $\mathbb{F}_q^n$ be the n-dimensional vector space over a finite field $\mathbb{F}_q$ with q elements and let it be equipped with an inner product. The hull of a linear code C is the intersection $C \cap C^\perp$, and its dimension plays an important role in constructing entanglement-assisted quantum error-correcting codes. If C is an $[n, k, d]_q$ linear code with dual distance $d^\perp$ and hull dimension h, then there exist entanglement-assisted quantum error-correcting codes with parameters $[[n, k-h, d; n-k-h]]_q$ and $[[n, n-k-h, d^\perp; k-h]]_q$. This construction is valid in the Euclidean case and also using the Hermitian inner product.

This research continues a recent enumeration of optimal linear codes with various hull dimensions. We use these computational results to obtain possible parameters of corresponding entanglement-assisted quantum error-correcting codes and compare them with known quantum coding bounds, including the EA Singleton, EA Hamming, and EA Plotkin bounds.

The Deficiency Spread of a Transitive Deficiency One Parallelism of $PG(5, 2)$

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Partial parallelisms of $PG(n, q)$ with exactly one missing spread are known as deficiency one parallelisms. Each deficiency one parallelism can be uniquely extended to a parallelism by adding the deficiency spread. A deficiency one parallelism is transitive if it is invariant under an automorphism group that fixes the deficiency spread and is transitive on the other spreads. Transitive deficiency one parallelisms in $PG(3, q)$ are thoroughly studied; the derived results are not relevant for greater dimensions $n > 3$ of $PG(n, q)$. Presently no transitive deficiency one parallelisms of $PG(5, 2)$ are known.

An automorphism of a transitive deficiency one parallelism of $PG(5, 2)$ fixes the deficiency spread, and the number of the remaining spreads is 30. Therefore the order of the full automorphism group of a possible transitive deficiency one parallelism of $PG(5, 2)$ should be divisible by 30. Previous work showed that there are two possibilities for the deficiency spread of a transitive deficiency one parallelism of $PG(5, 2)$, namely the two unique spreads, up to isomorphism, with automorphism group orders 5760 and 362880. The spread with the larger group is the regular spread in $PG(5, 2)$.

To search for transitive deficiency one parallelisms, all subgroups of order $30m$, $m > 1$, of these two groups have to be considered. In the present work we do this for the group of order 5760. We use the group theory functionality of GAP to obtain and investigate all subgroups of order $30m$ of the group of order 5760. Then we employ our own C++ program in an attempt to construct deficiency one parallelisms of $PG(5,2)$ invariant under these subgroups. We obtain no solutions and therefore claim that if there exists a deficiency one parallelism in $PG(5,2)$, its deficiency spread should be the regular spread of $PG(5,2)$, and its full automorphism group should be a subgroup of order $30m$ of the full automorphism group, of order 362880, of the regular spread of $PG(5,2)$.

On Schur Product of Algebraic Codes

Buket Özkaya[1*]
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In this talk, we study the Schur product of algebraic codes, with particular emphasis on cyclic codes. We begin by reviewing several known results on the Schur product of cyclic codes and the algebraic features that determine the parameters of the resulting codes. In particular, we discuss how the structure of cyclic codes influences the dimension, minimum distance, and other algebraic properties of their Schur products. We then present recent extensions and examples illustrating how methods developed for cyclic codes can be adapted to broader classes of algebraic codes. We then explain how these ideas can be extended to quasi-cyclic codes as well as to additive-cyclic codes, which are closely related to quasi-cyclic codes.

Some Asymptotically Optimal AAD Families

Talha Arıkan[1*]
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A family of k-dimensional subspaces of $\mathbb{F}_q^n$, which forms a partial spread, is called an L-almost affinely disjoint ($[n,k,L]_q$-AAD) family if each affine coset of a member of this family intersects with only at most L subspaces from the family. Recently, explicit constructions of AAD families have been attracting interest due to their various applications, such as constructing batch codes.

Known upper bounds describe the polynomial growth of the size of AAD families. It has been conjectured that there exist such families whose sizes attain this upper bound. Such families are called *asymptotically optimal* AAD families. This presentation reviews all asymptotically optimal AAD families designed with relevant parameters to date.

Binary Codes from Kronecker Product of Matrices

Nazahet Fellah[1], **Kenza Guenda**[2], **Ferruh Özbudak**[3*]

Key words: *Kronecker product of matrices; graph product; Paley-type bipartite graphs; self-dual codes; LCD codes*

Let C_1 and C_2 be two binary codes. Using the Kronecker product of their generator matrices, we construct new codes and give conditions on both C_1 and C_2 for which the constructed codes are self-orthogonal or LCD.

Recently, binary codes related to Paley-type bipartite graphs $P(q,k)$ have been studied by Fellah et al. In this paper, we extend the definition of the Paley-type bipartite graph to $q = 2$ and

$k = 1$ and describe the binary codes C and C' associated to the Cartesian and Kronecker product graphs $\Gamma_1 = P(q,k) \times P(2,1)$ and $\Gamma_2 = P(q,k) \wedge P(2,1)$, respectively. Furthermore, we see that the binary code obtained from the Kronecker product of two Paley-type bipartite graphs is LCD.

Plenary Lecture

The norm of Cauchy transform

Prof. Dr. David Kalaj[1*]
[1] **University of Montenegro**

We investigate the L^2 norm of the interior Cauchy transform and its connection with Laplacian eigenvalues on planar domains. In the case of circular annuli, symmetry and Fourier mode analysis lead to an exact norm formula in terms of the first mixed Neumann–Dirichlet eigenvalue, with extremizers given explicitly by Bessel functions. We further show that the quantity $2/\sqrt{\lambda_1(D)}$, associated with the first Dirichlet eigenvalue, is generally not sharp. In particular, for annuli and for simply connected domains other than the disk, the norm is strictly larger than this spectral threshold. Thus, the disk is the only domain for which the norm is determined exactly by the first Dirichlet eigenvalue.

Computer Algebra in Education (SS3)

Diagonalization and Change of Basis in Linear Algebra

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Key words: *Linear Algebra; Similarity; Basis change*

A common topic in a first course on Linear Algebra is diagonalization using eigenvectors. Typically the textbooks define "similarity" as $B = V^{-1}AV$ with the justification that the eigenvalues of A and B are the same. Later in the course (and in the text book) there is a brief section on change of basis. No connection is made between the topics. In this talk, I show the connection. It gives a geometrical justification for the similarity transform, and some nice plots. The change of basis connection extends to other diagonalizations met in more advanced courses: topics such as the SVD factors of a matrix.

Be careful when adding an integration constant

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Key words: *Indefinite and definite integrals; piecewise defined functions; continuous antiderivatives*

How many times have students in a calculus course been reminded to add a constant of integration at the end of an indefinite integral? Many times, no doubt. All the while, they often forget that the addition of this crucial constant of integration stems from the theorem stating that two antiderivatives of the same **continuous** function over an **interval** differ by a constant.

And all the while, they use integral tables that violate the aforementioned theorem and disregard the contributions of computer algebra systems from the last three decades!

They were told not to forget to add a constant of integration at the end of the calculation of an indefinite integral, but they rarely had the opportunity to calculate the constant of integration required to find a *continous* antiderivative of a piecewise continuous function. Or even to find the constant by which two antiderivatives of the same continuous function differ on a given interval.

We will attempt to show that if calculus textbooks include more examples of piecewise defined functions – these are so important in applied mathematics and in engineering –, the understanding of adding the constant of integration would likely be improved. The examples that will be presented were motivated by the following observation: several students taking a course in differential equations have difficulty finding the required constant of integration in a rectilinear motion problem. Especially if the given velocity of the object moving in a straight line is defined by a piecewise continuous function.

Computer Algebra Systems in Secondary Mathematics Education: The Role of GeoGebra CAS View and Generative Artificial Intelligence in Teaching and Learning Mathematics

José Manuel Dos Santos[1*] , Alexandre Trocado[2,3], Zsolt Lavicza[3]
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Key words: *Computer Algebra Systems; GeoGebra CAS View; Generative Artificial Intelligence; Secondary Mathematics Education; Instrumental Genesis; Multiple Representations; Assessment*

The integration of Computer Algebra Systems (CAS) in secondary mathematics education has been a subject of sustained inquiry for over four decades. CAS tools perform symbolic manipulation—equation solving, differentiation, factorisation—enabling learners to engage with exact mathematical representations. The emergence of GeoGebra, incorporating a dedicated CAS view since version 4.2, has transformed the accessibility of symbolic computation in secondary classrooms. Concurrently, generative artificial intelligence (GenAI) tools have introduced new capabilities for mathematical explanation, problem personalisation, and natural language scaffolding.

This paper presents a systematic analysis of the role of CAS in secondary mathematics education, with attention to GeoGebra's CAS view and its convergence with GenAI. The mathematical capabilities of current large language models are substantial but uneven. These systems can perform routine algebraic manipulations and generate code for mathematical software; however, their reasoning is fundamentally pattern-based rather than deductive, producing characteristic failure modes. This asymmetry defines a complementary model: GenAI is suited for natural language explanation, problem personalisation, and code scaffolding, whilst GeoGebra CAS provides reliable symbolic computation, algebraic verification, and dynamic visualisation.

Five conclusions emerge. First, CAS shifts instruction from procedural computation towards conceptual understanding. Second, GeoGebra's multi-view architecture enables distinctive multi-representational mathematical activity. Third, GenAI and CAS operate as complementary rather than competing tools. Fourth, pedagogical challenges necessitate graduated introduction and reasoning-focused assessment. Fifth, no controlled classroom study has yet evaluated the combined deployment of GeoGebra CAS view and GenAI in secondary settings, constituting the principal direction for future empirical research.

Life Sciences (SS2)

Evolutionary emergence of cooperativity in the game of Prisoner's Dilemma

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Key words: *Evolution of cooperation; Prisoner's Dilemma; Price equation; Fisher's fundamental theorem of natural selection*

Complex life would be impossible without cooperation at all levels of biological organization: genes cooperate to create regulatory networks and signaling pathways, cells become parts of multicellular organisms, animals participate in intricate cooperative arrangements both within and between species. Cooperation is ubiquitous in modern human societies and is credited with the initial rise and subsequent development of civilization. However, Darwinian selection is believed to promote selfish behavior - societies of cooperators are vulnerable to cheaters that ultimately take over, destroying cooperation in biological populations. This apparent paradox has puzzled evolutionary biologists for more than half a century. A model of the paradox is provided by the celebrated Prisoner's Dilemma, an elegant mathematical framework which favors cheaters, even though being a part of a cooperative society brings about greater overall rewards. Here we demonstrate that it is possible to achieve high levels of cooperativity in the game of Prisoner's Dilemma without introducing additional assumptions, commonly found in the literature, about genetic relatedness and spatial population structure. The only requirement is that the probability of cooperation vary depending on the nature of the opponent, instead of being blind to the opponent's physical appearance and patterns of behavior. Opponent-specific responses drive the population through a sequence of persistent, partially cooperative states - a plausible starting point for more advanced modes of cooperation observed in animal and human societies.

Topological Origin of Modularity in Chemical Reaction Networks

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In living cells, numerous chemical reactions are interconnected by sharing substrates and products, forming a reaction network. Various functions of cells emerge from dynamics of such interconnected system. Cells regulate amount of key chemicals by controlling amount/activity of enzymes, thereby achieving control of cellular functions. However, in such an interconnected system, can different chemicals responsible for different biological functions be controlled independently? We mathematically demonstrate that "modularity", where parts of a system are controlled independently of others, arises solely from network topology. We found that the qualitative response of chemical concentrations to changes in enzyme amount/activity are localized to finite ranges in a network, and each range is determined by a subnetwork called a "buffering structure". The bifurcation behaviors are also localized to finite regions within a network, and these regions are again determined by buffering structures. Using the cell cycle system as an example, we show that buffering structures actually exist in living organisms, performing important roles. In the cell cycle, the G1-S and G2-M transitions are strictly controlled by distinct protein complexes, requiring the activation of different complexes at different phases. Analysis of the cell cycle network revealed that the two complexes belong to different buffering structures. Moreover, by

comparing theoretical predictions with experimental verification, we theoretically predict the necessity of an unknown reaction and experimentally confirm it.

Structural identifiability of bacteria co-culturing models

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In this work we created a full dataset of specific class of two bacteria co-culturing models and performed different structural identifiability algorithms on them. This is the first large scale and systematic benchmarking for structural identifiability to the date with more than 100,000 models, additionally all the models are coming from a single application. We identified linear relations that enable a reduction in the dimensionality of inputs to the Gröbner basis computations arising in the algebraic approach to structural identifiability and thus improving previous implementations of this algorithm.

Polyhedral Geometry of Max-Linear Bayesian Networks

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Max-linear Bayesian networks (MLBNs) are a class of directed acyclic graphical (DAG) models which are of interest to statistics and data science due to their relevance to causality and probabilistic inference, particularly of extreme events. The conditional independence (CI) relation of a max-linear Bayesian network is fully determined by the combinatorics of critical paths in the underlying weighted graph. We call these CI structures maxoids, and prove that every maxoid can be obtained from a transitively closed weighted DAG. We show that the stratification of generic weight matrices by their maxoids yields a polyhedral fan which is the normal fan of the Minkowski sum of the Newton polytopes of path polynomials in the graph. The derived constructions form the basis for computational approaches to problems of enumeration and conditional independence implication. This is joint work with Tobias Boege, Kamillo Ferry, and Ben Hollering.

Computer Algebra to Study Bifurcations in Chemical Reaction Networks

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Chemical reaction networks (CRNs) are a central component of many models in mathematical biology. One way to make sense of the dynamical behaviours of CRNs across parameter space is by studying their bifurcations. Computer algebra has been highly successful in helping us study bifurcations in small reaction networks. However, as network size increases, the complexity of the computational problems grows rapidly. I'll discuss some recent theoretical results which help us decide on the capacity of CRNs for various bifurcations, and simplify the computations needed, even in some classes of larger networks.

Plenary Lecture

Computing with Symbolic Domains

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Key words: *Symbolic domains, Symbolic exponents, Symbolic matrix structure, Hybrid sets*

We outline computational methods for algebraic quantities whose sizes or shapes are given symbolically. By this we mean the usual objects of algebra, but where certain parameters, shapes or sizes are given by symbols or formulæ. These frequently arise in hand computation without a second thought: a simple example is when one writes a "polynomial" in the form

$$p = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0.$$

This expression is *not* actually a polynomial and is not subject to the powerful algorithms of computer algebra since the degree and number of terms are not fixed. Nonetheless, we expect to perform computations such as

$$x^{2m} - y^{2n} = (x^m + y^n)(x^m - y^n) \quad \text{and} \quad \frac{\partial}{\partial x}\left(x^{2m} - y^{2n}\right) = 2m x^{2m-1}.$$

We have argued [7, 9] that examples such as these show a gap between the algebraic structures used by *computer algebra* and the term algebras of *symbolic computation*. To bridge this gap we must treat symbolic domains algebraically. Here we provide an overview of some work toward this goal.

A second type of example arises in integration over domains given symbolically. One may wish to express relations such as

$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega,$$

where the region Ω and its boundary $\partial\Omega$ are specified by formulas rather than explicit decompositions. In these settings, a direct computational approach requires partitioning the domain into many disjoint regions, often leading to a combinatorial explosion. By contrast, informal mathematical reasoning proceeds by treating the domain symbolically, relying on cancellation across overlapping regions and on implicit inclusion–exclusion principles.

In our review, we first consider polynomials with symbolic exponents. In this setting, operations such as greatest common divisor computation, factorization, and functional decomposition must be reformulated to operate on parametric exponent structures. Rather than expanding expressions for specific values, these algorithms treat exponent relations symbolically, enabling uniform computation over entire families of polynomials. This leads to methods that extend classical algebraic algorithms while avoiding unnecessary case distinctions [8, 10, 11]. The key observation here is that $x^{\binom{n}{0}}$, $x^{\binom{n}{1}}$, $x^{\binom{n}{2}}$, *etc.*, are algebraically independent and that by taking exponents to be integer-valued polynomials in a binomial basis, the fixed divisors of the exponent polynomials are given by their content (the GCD of their coefficients). For example $x^{n(n-1)} = x^{2\binom{n}{2}}, n \in \mathbb{Z}$, is always a square.

We then consider matrices with symbolic internal structure, including block and banded forms where dimensions are given by symbolic expressions. Here, the key issue is that matrix structure depends on relationships between symbolic parameters, and naive approaches require splitting into many cases depending on inequalities between dimensions. A sequence of works has shown how to treat such matrices as first-class objects, supporting both arithmetic and logical reasoning over generic cases. This involves representing structural conditions symbolically and propagating them through computations, rather than enumerating all configurations [4, 5, 6].

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These developments lead naturally to the question of how to represent symbolic domains themselves. The central contribution we highlight is the use of hybrid sets for this purpose. Hybrid sets generalize multisets by allowing negative multiplicities, providing a natural algebraic framework for inclusion and exclusion within symbolic regions. This makes it possible to combine overlapping regions directly and to represent cancellations explicitly, without first reducing to disjoint cases [2].

When applied to symbolic block matrices, hybrid support sets provide a uniform representation of matrix structure across all configurations. Instead of performing separate computations for each case, one works with algebraic combinations of regions, allowing common structure to be shared. Most significantly, this approach reduces the expression complexity of computations from exponential in the number of symbolic partitions to linear, replacing explicit case splitting by symbolic combination of regions.

Beyond matrices, hybrid sets provide a unifying representation for a broad class of problems involving domains defined by formulas. In particular, they give a computational realization of generalized inclusion–exclusion principles and support compact representations of symbolic domain decompositions. This allows operations such as integration, restriction, and boundary formation to be expressed directly at the symbolic level, without expansion into explicit pieces [1, 3].

Taken together, these developments illustrate a broader shift from purely algebraic computation toward symbolic computation in which the defining structure of expressions is explicitly represented and manipulated. By working at the level of structure rather than instances, it becomes possible to avoid combinatorial explosion and to treat entire families of objects uniformly. This perspective and its algorithmic consequences are surveyed in [12].

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General Session (SS1)

Evaluating Large Language Models on Symbolic Reasoning Tasks

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Large language models (LLMs) have demonstrated strong capabilities in natural language generation; however, their reliability in tasks requiring structured and symbolic reasoning remains an open research question. This paper explores how LLM behaviour can be systematically evaluated when deployed in retrieval-augmented generation (RAG) systems operating over domain-specific knowledge bases. In particular, we focus on symbolic reasoning tasks that require the consistent use of structured information rather than purely generative language abilities. To investigate this, we design a RAG pipeline built on a corpus of sustainable energy data, representing a knowledge domain where accuracy and traceability are essential. The system retrieves relevant documents and uses an LLM to generate responses that combine retrieved evidence with reasoning steps. We then conduct experiments by varying several generation parameters (e.g., temperature, top-k, top-p, typical-p), in order to observe how decoding strategies influence the model's reasoning behaviour and output characteristics. The evaluation framework examines multiple dimensions of output quality, including factual accuracy, relevance to the query, readability, and consistency with retrieved evidence. Where possible, assessments are complemented with domain-expert ratings to capture qualitative aspects of reasoning and explanation. Particular attention is given to the trade-offs between determinism and diversity; for example, whether lower temperature improves factual precision while reducing interpretative depth, or whether adjusting typical-p can mitigate hallucinations in complex reasoning scenarios. The study contributes insights into how generation parameters influence the reliability of LLM-driven reasoning within applied RAG systems.

Resolving the Remaining Cases of Carlitz-Extensiveness over Finite Fields using SageMath

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The primitive normal basis theorem asserts the existence of elements in a finite extension of finite fields which simultaneously generate the multiplicative group and a normal basis over the base field. This work deals with a generalisation of this problem.

Hsu and Nan proposed a Carlitz-module analogue of this theorem, replacing primitive normal elements by primitive generators for finite Carlitz modules. Their work establishes the desired existence result with one confirmed exception, $(q, n) = (2, 2)$, and a finite, effectively computable list of further possible exceptional cases. Hachenberger subsequently placed this question into the broader framework of primitive generators for cyclic vector spaces over Galois fields; in particular, his work resolved 41 of the 63 undecided pairs left by Hsu and Nan. In the concluding remarks of Hachenberger's paper, computations by Gruber using SageMath further reduce the Carlitz-extensiveness problem to six remaining undecided pairs. The present work resolves these remaining cases.

The talk explains the computational part of this final step. We present a fully reproducible SageMath implementation designed to handle the six undecided parameter pairs left after the reductions of Hsu-Nan, Hachenberger, and Gruber. The implementation combines finite-field techniques, including computations with cyclotomic cosets, with projective-space methods and minimal-polynomial-based verification procedures for the relevant generator properties. Together, these computational results establish the Carlitz-module analogue of the primitive normal basis theorem apart from the single exceptional pair $(q, n) = (2, 2)$ identified by Hsu and Nan. In particular, the resolution of the six remaining pairs confirms that primitive Carlitz generators exist for all other finite field extensions.

Nonabelian Cohomology and Classifying Spaces of Crossed Complexes

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Key words: *Principal bundles; Nonabelian cohomology; Crossed complexes; Classifying spaces; Geometric realization*

Two of the most fundamental approaches to the study of principal bundles are through cohomology and classifying spaces. In this talk, we compare nonabelian cohomology with coefficients in a topological crossed complex $\mathcal{G}$ with homotopy classes of maps into its classifying space ($B\mathcal{G}$). To establish an equivalence between these two perspectives, we use the topological group structure arising from the classifying space of a topological simplicial group, together with the fact that geometric realization preserves short exact sequences. This gives a bridge between nonabelian cohomological methods and homotopy-theoretic classification results for principal bundles and their higher analogues.

Computer Algebra in Education (SS3)

From Curricular Signals to Exploratory Tasks: A Curriculum and Pedagogical Design Case Study for Integrating CAS in Upper Secondary Education

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Key words: *Computer Algebra Systems; GeoGebra CAS; Python/SymPy; Upper Secondary Education; Curriculum Analysis; White-Box/Black-Box; Computational Thinking; Pedagogical Design*

How can national curricula that generically endorse "the use of technology" be converted into concrete pedagogical frameworks for integrating Computer Algebra Systems in upper secondary mathematics? This talk addresses this question through a curriculum and pedagogical design case study grounded in the Portuguese *Aprendizagens Essenciais* (Essential Learning Standards) for Mathematics A — the science-track upper secondary course (grades 10–12, 2023 revision).

A systematic document analysis of the three curricular documents, guided by five analytical dimensions, reveals four curricular signals that jointly support a pedagogically grounded CAS integration. The first is an *implicit legitimation* of symbolic computation. The second is a *concentration of CAS potential on core topics*: functions, derivatives, sequences, analytic geometry, and complex numbers. The third is a *vertical anticipation chain*: CAS-mediated exploration at each grade level opens mathematically coherent windows onto concepts from subsequent years. The fourth is *Python and Computational Thinking as a gateway.*

Although rooted in the Portuguese case, the contribution is designed for international transferability. The talk offers an analytical grid for reading any curriculum in terms of CAS integration potential, a set of explicit pedagogical design principles, and a staged framework adaptable to other national contexts.

Tuesday, June 2 — Room 114

Group Theory (SS5)

Classification of Artin groups that admit retractions to parabolic subgroups

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An Artin group is a group generated by a finite set of (standard) generators subject to relations of the type $sts\cdots = tst\cdots$, where both sides of the equality have the same number of letters. A (standard) parabolic subgroup is a subgroup generated by a subset of generators, which is again an Artin group. We say that an Artin group admits a retraction onto a parabolic subgroup if there exists a retraction homomorphism from the Artin group onto that parabolic subgroup. In this work, we classify the Artin groups admitting such retractions. We also prove that whenever such a retraction exists, there always exists a retraction sending standard generators to standard generators or identity. This is joint work with B. A. Cisneros de la Cruz, I. Foniqi, and L. Paris.

Artin groups and their isomorphism problem

Giovanni Sartori[1*]
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In this talk, I will introduce the isomorphism problem for Artin groups, as well as isomorphism invariants that can be read from the defining graph. We will specifically focus on the girth of the defining graph: I will give an algebraic characterisation of it, showing that it is an isomorphism invariant.

JSJ decompositions of Artin groups

Oli Jones[1*]
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Artin groups are a rich class of groups generalising Braid groups and closely related to Coxeter groups. In this talk I will present a characterisation of which Artin groups split over infinite cyclic subgroups, leading to an explicit JSJ decomposition of Artin groups over infinite cyclic subgroups. I will then discuss how tools related to the JSJ decomposition can be used to extract information about the (outer) automorphism groups of certain Artin groups, such as their finiteness properties. The talk is partially based on joint work with Giorgio Mangioni and Giovanni Sartori.

Parabolic Subgroups and the Word Problem in Virtual Artin Groups

Federica Gavazzi[1*]
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Virtual Artin groups were introduced by Bellingeri, Paris, and Thiel in 2023 as a generalization of virtual braids. These groups possess a rich structure, simultaneously encoding features of classical Artin groups, Coxeter groups, and root systems. For these groups, which are defined by a presentation with generators and relations, a parabolic subgroup is a conjugate of a subgroup generated by a subset of the generators. For classical Artin groups, it remains an open problem whether the intersection of two parabolic subgroups is again a parabolic subgroup. In this talk, we discuss the word problem and the notion of parabolic subgroups for virtual Artin groups, highlighting their connections with the associated classical Artin and Coxeter groups.

A finitely presented group is almost planar if and only if it is virtually planar

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A group is planar if it admits a Cayley graph that is a planar graph. While the class of planar groups is hard to describe, it is not the case for the class of *virtually planar* groups, namely groups with a finite-index subgroup that is planar, the latter being precisely those groups which are virtually free products of free and surface groups. It is straightforward to see that if a group is virtually planar then it is *almost planar*, meaning that some/every Cayley graph of it can be drawn in the plane so that each edge crosses at most k other edges. We show that the converse is also true. This is joint work with John Mackay and Joseph MacManus.

Thursday, June 4 — Room 153

General Session (SS1)

Strongly Orthogonal Subsets in Exceptional Root Systems: Theory and Computation

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Key words: *root systems; graph theory; sunflowers*

We study a family of graphs whose vertices are labelled by sums of k-element strongly orthogonal subsets (SOS) of a root system R, with edges between vertices whose difference is itself a vertex. The maximum clique size in such a graph, denoted $\mu_k(R)$, is the central combinatorial quantity of interest. Previous work by the authors and Gashi established Erdős–Ko–Rado (EKR) type results for the classical Type A root systems. Here we extend this programme to the five exceptional root systems G_2, F_4, E_6, E_7, and E_8.

We describe both the theoretical framework and the computational methodology underlying our results. The SOS graphs for exceptional types are constructed by two methods independently. Firstly, a Python/SageMath pipeline enumerates all strongly orthogonal subsets of each size k, builds the corresponding graph, and computes standard invariants — vertex and edge counts, degree sequences, and connected components. Clique numbers are determined using the `cliquer` package, and the action of the Weyl group $W(R)$ on the graph is exploited via orbit-counting arguments to give exact enumeration of maximum cliques and maximum sunflower cliques across all valid k. Secondly, using the *Claude Code* AI framework.

In this talk we give an introduction to the problem and discuss the hurdles on the computational side as well as the AI-assisted computations.

Efficient Algorithms for Quaternion Tensor Completion

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Key words: *Quaternion tensor completion; QR decomposition; $L_{2,1}$-norm; Video inpainting*

Tensor completion is a fundamental task in computer vision and image processing, with wide-ranging applications from recommendation systems to medical imaging. Prevailing methods, predominantly based on the singular value decomposition (SVD) of real-valued tensors and nuclear norm minimization, face significant challenges: they often fail to preserve the intrinsic correlation among color channels and lack robustness against variations across video frames. Moreover, their reliance on computationally intensive SVD operations limits their scalability and practical utility for large-scale data. To address these limitations, we design the accelerated SVD decomposition algorithm: QTSVD-QR, and propose a novel quaternion tensor (matrix) completion method that utilizes QR decomposition and $L_{2,1}$-norm minimization (QTLNM-TQR), which can effectively balance model generalization ability with computational efficiency, resulting in a notable enhancement in completion performance. Numerical experiments conducted on color images, videos, and PET-CT provide compelling evidence of the proposed method's effectiveness.

Computer Algebra in Education (SS3)

Integration of Computer Algebra Systems and AI in HTML-based Mathematics Materials using KeTCindyJS

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Key words: *KeTCindyJS; KeTCindyChatBot; Maxima; Algebrite; Nerdamer;*

The author has developed HTML-based teaching materials using KeTCindyJS, and in 2025 demonstrated that integrating KeTCindyJS with Maxima enables mathematically accurate and visually clear materials based on symbolic computation. However, this approach requires pre-computation in a local environment and is limited to predefined functions, restricting flexible interaction in web browsers. To address this issue, Algebrite, which enables symbolic computation within a browser, is introduced to construct an environment that supports arbitrary functions and equations. In addition, to reduce the difficulty of developing teaching materials using CindyScript, we are developing KeTCindyChatBot, and, as an application of the materials, an expression input/output system equipped with an automatic grading function using Nerdamer.

By integrating KeTCindyJS with Algebrite, we enabled the development of HTML-based teaching materials that allow symbolic computation in a web browser. This allows learners to input arbitrary functions and equations and perform symbolic operations interactively, thereby promoting exploratory learning. As one application of the developed HTML-based teaching materials, the author, together with students, is developing an e-learning system. The submitted expressions are converted and processed using Nerdamer, a JavaScript-based computer algebra system. By leveraging symbolic computation, the system evaluates the equivalence between the submitted expressions and the expected answers.

By combining symbolic computation with interactive visualization, it becomes possible to create mathematically accurate and intuitive teaching materials. Future work includes improving the accuracy and functionality of KeTCindyChatBot, as well as evaluating its effectiveness through educational practice.

Envelopes of Circles Centered on a Cubic-Quadratic: A Dynamic Exploration of their Topology and Singular Locus

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This study investigates the envelopes of a 1-parameter family of circles with constant radius r (for various values of r), centered on the cubic-quadratic curves $\mathcal{Q}_a$, defined by $y^2 = x^3 + ax$, where $a \in \mathbb{R}$. We analyze how the topology of the envelope changes as a function of a and r. Networking Dynamic Geometry Systems (DGS) with Computer Algebra Systems (CAS) and Generative AI, we identify complex singular structures (cusps, crunodes, and self-tangencies). This work enhances the necessity to develop automatic communication between between DGS and CAS to validate conjectures with rigorous algebraic proof.

ChatGPT as an instrument for enhancement of learners' interest in undergraduate mathematics

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Key words: *ChatGPT, instrument, undergraduate mathematics*

We are facing reality where Tradition welcomes Technology and Technology congratulates Tradition. Both can help overcome weak points in mathematics education and in learners' resulting mathematics knowledge and skills.

ChatGPT has at disposal great amount of knowledge, paradigms and experience of the humanity. The software "behind" the ChatGPT makes it possible to extract the necessary portions of existing information and provides in a noticeable way answers of questions and solutions of problems. It is quite understandable why one can trust ChatGPT competency. Obviously we have not to compete with ChatGPT but to collaborate and cooperate effectively with ChatGPT in order to facilitate the teaching and learning of mathematics.

It is important to consider the balance between application of generative AI in real educational process and the human experience without application of AI. In case of mathematics education it is necessary.

AI is a good opportunity to give new life of our good old approaches to facilitate learning theoretical facts and solving problems effectively. Up to now they are described only in learning resources like textbooks. Through ChatGPT they will be available to a larger audience.

As the answer of existing versions of ChatGPT depends on the preciseness of the question, it is necessary to enhance learners' capability. And to achieve this, learners' comprehension of knowledge is decisive. ChatGPT environment could be combined with CAS (Computer Algebra System) environment. The synergy effect of this could be valuable.

Learning outcomes, educational goals, exam questions and assessment of learners' knowledge and capabilities in doing mathematics are to be reconsidered in case of ChatGPT environment. Examples from undergraduate mathematics are considered.

Intersections of a Torus with Cubic Surfaces via Planar Section Reduction

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Key words: *Torus; Cubic Surfaces; Algebraic Intersection; Bézout's Theorem; Horizontal Sections; Symbolic Computation; Dynamic Geometry; GeoGebra*

This paper studies the intersection between a torus of revolution and two related families of cubic surfaces: the structured class defined by a product of three affine linear factors perturbed by a constant, and the full cubic family written with twenty independent coefficients. The analysis is organised by horizontal sections $z = k$, which reduce the spatial problem to a family of planar problems between a cubic curve and the two circular components of the torus section. The investigation has a twofold purpose: first, to organise the mathematical analysis of torus–cubic intersections through section reduction; secondly, to extend the analysis from the factorised cubic family to the general cubic case and to document the results through paired symbolic and dynamic representations.

The horizontal-section method provides an effective framework for studying torus–cubic intersections. For the torus, it yields an exact decomposition into two concentric circles. For the cubic, the same principle produces a planar cubic regardless of whether the spatial equation is factorised or fully general. The extension from the structured to the general family preserves the section method, the Bézout bound, and the symbolic–dynamic complementarity, whilst changing the geometric interpretation. The worked example with twenty distinct integer coefficients demonstrates that maximal real intersection behaviour persists in the general regime, confirming that the phenomena observed in the planar reduction belong to the interaction between cubic section families and toroidal section structure rather than to factorisation alone.

Solving Wasan Problems on Malfatti Circles via the *mnr* Method: Human-AI Collaborative Automated Derivation using KeTCindy

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Key words: *Malfatti Circles; Wasan; KeTCindy; mnr Method; Gemini*

During the Edo period (from the 17th to the mid-19th century) in Japan, interactions with Europe were strictly limited due to the national isolation policy (*Sakoku*). As a consequence, a unique, indigenous mathematical tradition known as *Wasan* flourished independently. *Wasan* mathematicians established advanced methods for solving complex algebraic equations, which enabled them to solve a wide variety of intricate geometric problems. One notable example is the problem of Malfatti circles.

While *Wasan* mathematicians solved these problems relying on their extraordinary intuition and computational skills, this paper aims to solve the Malfatti circles problem from the Myojo-rinji Temple *Sangaku* using modern computer algebra systems (CAS). Specifically, formulating a system of equations based directly on the tangency conditions of the circles typically introduces irrational expressions, quickly leading to computational failure. To overcome this, we employ a method that expresses various geometric quantities as rational expressions in terms of the tangents of the two half base angles, $m = \tan(B/2)$ and $n = \tan(C/2)$, along with the inradius r. This approach is referred to as the *mnr* method.

By integrating the historical wisdom of *Wasan*, the geometric algebraic formulation of the *mnr* method, and the computational power of KeTCindy and Maxima, we successfully re-derived Kazuhide Omura's beautiful 1841 formula. A crucial factor in this success was the collaborative problem-solving with generative AI. Gemini acted not merely as a code debugger, but as a strategic partner in navigating algorithmic limitations. This study highlights that human-AI collaboration holds immense potential for the future of computer algebra research and mathematics education.

Ovals and pedal curves: space trajectory models and genealogy of plane curves

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Key words: *Pedal curves; Networking with technologies; Space trajectories*

The general media frequently report on the launches of satellites and space probes, often accompanying these reports with graphics illustrating various orbital trajectories. These "general" representations serve as a rich foundation for sophisticated mathematical activities with a clear interdisciplinary STEAM approach. By networking between the graphical features of Dynamic Geometry Systems (DGS) and the symbolic strength of Computer Algebra Systems (CAS), we create an environment for exploration, conjecture validation, and rigorous modeling. Following Kepler, central force motion is modeled using ellipses. In this framework, a space probe's trajectory is represented by arcs of ellipses; changes in orbital geometry, specifically eccentricity, are induced by the activation of engines.

Historically, before the widespread adoption of Kepler's model, Giovanni Domenico Cassini proposed an alternative: the *Cassini ovals* (or spiric curves). Their bifocal definition is analogous to that of the ellipse. While an ellipse is a quadratic curve, a Cassini oval is determined by a quartic polynomial. It is an oval for sufficiently large r; otherwise, the spiric curve appears as two disjoint closed components that remain algebraically irreducible, the limiting case being a Bernoulli lemniscate.

Historically, another closed curve—the *Hippopede*—was conjectured as a model for planetary or satellite motion. This curve can be generated as the **pedal curve** of an ellipse with respect to its center. The pedal of an ellipse with respect to its center is known as a *Booth curve* or the *Hippopede of Proclus*. Distinguishing these curves from ellipses often requires high-precision visualization or symbolic analysis. The modeling of these curves utilizes *GeoGebra-Discovery*. While the numerical `Locus(B,A)` command provides immediate graphical feedback, the symbolic `LocusEquation(B,A)` command is essential for deriving the underlying polynomial equations.

Friday, June 5 — Room 153

General Session (SS1)

Polygraphic resolutions for plactic monoids: revisiting my thesis with new tools

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Plactic monoids are rich combinatorial objects, showing up as models of mathematical objects in various fields. In particular a plactic monoid can be seen as a model of the representation theory of complex Lie algebras. These monoids admit presentations given by generators and oriented relations and which satisfy two non-degeneracy properties: termination and confluence. Such a presentation allows the construction of a polygraphic resolution which may be seen as an analogue of an infinite simplicial complex sitting on top of the monoid, and containing higher dimensional information about it. In my thesis and subsequent work it has been shown that if such a presentation commutes with the weight-shifting operators from Lie algebras, then the 3-cells of highest weight of the polygraphic resolution determine the structure of all of its 3-cells. Moreover, for the classical types A and C we have introduced combinatorial objects via which the boundaries of all the 3-cells can be computed.

In this work in progress, we aim to expand these results to other classical types and to higher dimensional cells through the use of computational tools and AI.

Residual Centering and Error Bounds for Convex Approximation of Mixed-Integer Recourse

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Key words: *stochastic mixed-integer programming; bounded variation; Vitali variation; residual error bounds*

We study convex approximations of mixed-integer recourse functions in two-stage stochastic programming. For first-stage decisions, the relevant error is the signed expected residual between the approximation and the integer-recourse value. We identify residual conditions that imply deterministic expectation-error bounds on bounded uncertainty boxes. The central condition is coordinate slice-centering, under which integration by parts yields anisotropic total-variation and Vitali-type mixed-derivative bounds in terms of the density. Exact centering, however, may be incompatible with convexity. We therefore introduce coordinate projectors that decompose an arbitrary residual into a centered component and a slice-mean defect. The resulting defect adjusted bounds support a max-affine convex fitting framework that penalizes and audits the quantities appearing in the theory.

Stone Duality Proofs for Colorless Distributed Computability Theorems

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In a distributed system, several processes attempt to coordinate through some form of communication in order to solve a task. Characterizing task-solvability for general models of communication has been a central question in distributed computing since its foundation. In 2004, Herlihy and Shavit [4], as well as Saks and Zaharoglou [5], building on the work of Borowsky and Gafni [1,2], were awarded the Gödel prize for showing that central problems in distributed computability can be understood and solved using topology.

Indeed, the possible epistemic states of a distributed system fit nicely into finite combinatorial objects called simplicial complexes, which have a clear topological interpretation. Concretely, such an object consists of all possible global states of the system, glued along shared subsets. Each global state in a distributed system is a collection of local states. This not only represents the combinatorics of possible configurations, but also encodes the lack of global information available to each process: a process in a certain local state cannot distinguish between two global states if its local state lies in their intersection. Simplicial complexes, which can also be thought of as spaces built by gluing together points, lines, triangles and their higher dimensional analogues, thus precisely encode the epistemic ambiguities of the system.

A typical distributed problem, a task, is for example presented as a triple (I, O, Δ) where I and O are simplicial complexes, and Δ a relation between them. Furthermore, at least for some tasks, it is known that continuous maps between the geometric realizations of associated simplicial complexes classify which tasks can be solved by the protocol. While many generalizations and applications of this topological approach have been developed, still to this day, the direct connection between distributed computing and continuous maps between geometric realizations was unclear. Here, we identify spectral spaces, arising as limits of the finite combinatorial topology models of distributed computing, as the underlying phenomenon behind this remarkable connection. Our work shows that this topological characterization is in fact a result of Stone duality, which is a well established tool for such correspondences in computer science. This additionally allows us to obtain a version of the original result which applies to a much wider class of protocols.

In short, our main contribution is the identification of spectral topology as a natural and fruitful extension of the simplicial semantics for distributed computing. This perspective allows us to generalize known colorless topological computability results to any round-based, full-information model of computation. We achieve this by encoding such a protocol as an endofunctor Π on the category of simplicial complexes. Using this, we associate a spectral space $\Pi^\infty(I)$ to any input complex I, which can be described abstractly in terms of a projective limit, but also concretely as a space of sequences. These spaces characterize computability, as is shown in our main theorem, which states that a protocol Π solves a task (I, O, Δ) if, and only if, there exists a spectral map from $\Pi^\infty(I)$ to the output complex O which respects the specification Δ. For a more developed description of our techniques and results, see the pre-print [3], which has recently been accepted at ICALP'26.

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A Journey Through Decision Problems in Monoids and Groups

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The word problem for one-relator monoids remains one of the long-standing open questions in combinatorial algebra. One approach to this problem is through the study of related decision problems, particularly the submonoid membership problem, in both monoids and groups. In this talk, I will explore how these problems are connected, drawing on classical work of Sergei Adian and Victor Guba. I will also highlight the role of embeddings of trace monoids, which provide useful insight into the underlying structure of these questions. More broadly, the talk offers an overview of decision problems in groups and monoids, emphasizing key results on decidability and the contrasts between the two settings.

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